

# A new optical measurement system for determining the geometrical errors of rotary axis of a 5-axis miniaturized machine tool<sup>†</sup>

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## Abstract

Machine tools have greatly improved in recent decades. Among them, miniaturized machine tools (mMTs) that have advantages in terms of reduced energy consumption, space requirements, costs, and other resources are becoming more and more popular in the area of micro scale parts manufacturing. However it is exceedingly difficult to calibrate an mMT due to its small size. This paper proposes a novel method for identifying the geometrical errors of a rotary axis of miniaturized 5-axis machine tools using two position sensitive detectors (PSDs) and a laser diode. This work not only reduces the complication of the system structure and setup but also solves the problem of assembly. We propose a method to determine errors based on the geometrical position of PSDs, laser beam path, and its read-out signals after each angle of rotation. The homogenous transformation approach is used to find the individual error components. A system sensitivity analysis is also presented in this paper.

*Keywords:* Geometrical error; Homogenous transformation; Laser diode; Miniaturized machine tool; PSDs

## 1. Introduction

Since the development of the first machine tool in the beginning of the 18th century, machines have become more precise and convenient. However the requirement of micro-scale manufacturing, the limitation of materials, and the conservation of power and area have led to the development of miniaturized machine tools (mMTs) [1-3]. The calibration of an mMT is exceedingly difficult due to the requirement of high accuracy combined with a rotary stage that is less than 100 mm in size. Hence it is necessary to design a compact measurement system that can be used with the small stages of mMT.

Common measurement systems for measuring geometrical errors of rotary axis of machine tool include laser interferometer and double ball bar system. Though the latter is economical, due to its size it cannot be used to measure the geometric errors of mMTs. Laser interferometer is widely used to measure geometrical errors due to its high accuracy. However, for a rotary axis, it can only measure the angular position accuracy. Moreover, it is not economical. Recently, different methods for determining the rotational deviation were developed. In

1998, Suh et al. developed a measurement system using auto-collimator and linear variable differential transformer (LVDT) to measure the six geometrical error components of a rotary axis [2]. In 2004, Liu et al. completed a measuring system using diffraction grating, laser diode, and position sensitive detectors (PSDs) for measuring error motions and angular indexing of an indexing table [3].

In this paper, we propose a new measurement system to separately measure the six geometric errors of a rotary axis. It is economical, simple to use and the results are easily obtained. The system consists of a laser diode, two PSDs and beam splitters, and a turning mirror. The laser beam from laser diode is split by the first beam splitter. In the Step 1, the transparent beam is used while the reflected beam in the perpendicular direction (Beam A) is blocked. The transparent beam is reflected by a turning mirror and reaches PSD1 in a tilted direction.

In the Step 2, the shutter blocks the transparent beam and Beam A is used. This beam is again split by the second beam splitter before reaching PSD1 and PSD2 which are orthogonally fixed on the rotary stage. The signals from two PSDs are then recorded thrice, which include the information about the six geometrical errors. Therefore using the homogenous transformation matrix, the interrelation between the signals from PSDs and the six geometrical errors is derived.

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**2. System configuration and measuring procedure**

**2.1 System configuration**

The measurement system includes two modules, as shown in Fig. 1. Module 1, with laser diode, beam splitter 1, and reflector is fixed outside the stage, while Module 2, with beam splitter 2 and the PSDs is fixed on the stage. Fig. 1(a) shows the configuration of Step 1. The laser diode is fixed horizontally and PSD1 is fixed on the rotary axis. The transparent beam is reflected by the reflector and strikes PSD1 in a tilted direction. Fig. 1(b) shows the configuration of Step 2. In this step, the two split laser beams perpendicularly strike PSD1 and PSD2. The data is captured at the same rotated angle in both steps.

**2.2 Procedure**

In Step 1, the two signals from PSD1 include the information of five geometrical error components. The information of the sixth error is recorded by the two signals from PSD2. On the other hand, the PSD1 gives two more signals with information of two error components. The six relationships between the geometrical errors and the signal of PSDs generate a system of six equations, which are sufficient to calculate all error component. In a circular motion (shown in Fig. 2), each

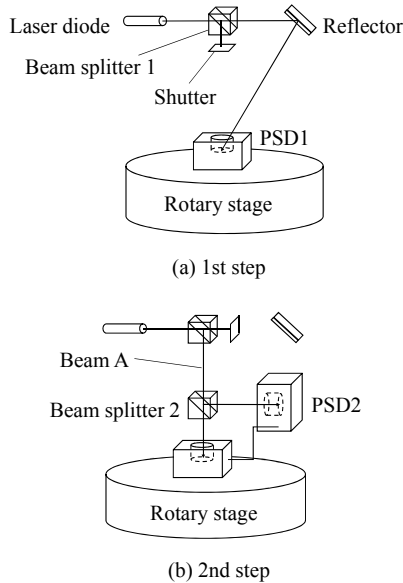


Fig. 1. Measurement system principle.

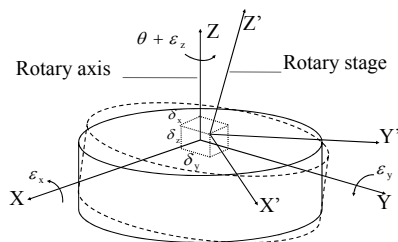


Fig. 2. Rotary axis geometric errors.

body has six degree-of-freedom (DOF) geometric errors: three displacement errors (two radial and one axial error) and three rotational errors (two tilt and one angular error). These error motions are combined into a homogenous transformation matrix, *R*, as shown in Eq. (1).

$$R = \begin{pmatrix} \cos\theta - \sin\theta \epsilon_z & -\sin\theta - \cos\theta \epsilon_z & \epsilon_y & \delta_x \\ \sin\theta + \cos\theta \epsilon_z & \cos\theta - \sin\theta \epsilon_z & -\epsilon_x & \delta_y \\ \sin\theta \epsilon_x - \cos\theta \epsilon_y & \cos\theta \epsilon_x + \sin\theta \epsilon_y & 1 & \delta_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (1)$$

where *Z* is the axis of rotation,  $\delta_x$  and  $\delta_y$  are the radial displacement errors,  $\delta_z$  is the axial displacement error,  $\epsilon_x$  and  $\epsilon_y$  are the tilt errors,  $\epsilon_z$  is angular error and  $\theta$  is the rotated angle. After each rotation of an angle of  $\theta$  degree, using *R*, the coordinates of spotted point of laser beam on PSD are determined as functions of  $\theta$  and geometrical error components. These coordinates are transformed to PSD coordinates and compared to the value of *X* and *Y* at the output of this PSD. The six equations are thus generated.

**2.2.1 Step1**

The coordinate frame of PSD and the reference coordinate are defined as shown in Fig. 3. The (*X*,*Y*,*Z*) is the reference coordinate system and (*X<sub>p</sub>*,*Y<sub>p</sub>*,*Z<sub>p</sub>*) is the PSD coordinate system. The *P* value is the spotted point of laser beam on the PSD surface and the read-out signal from PSD shows the values of point *P*(*x<sub>p1</sub>*,*y<sub>p1</sub>*,*z<sub>p1</sub>*) coordinates in the (*X<sub>p</sub>*,*Y<sub>p</sub>*,*Z<sub>p</sub>*) coordinate system. It is clearly shown that *z<sub>p1</sub>* is equal to zero. The position of *P* in reference coordinate is determined by using error matrix *R*.

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \delta_x + (\cos\theta - \sin\theta \epsilon_z)x_{p1} - (\sin\theta + \cos\theta \epsilon_z)y_{p1} \\ \delta_y + (\sin\theta + \cos\theta \epsilon_z)x_{p1} + (\cos\theta - \sin\theta \epsilon_z)y_{p1} \\ \delta_z + (\sin\theta \epsilon_x - \cos\theta \epsilon_y)x_{p1} + (\cos\theta \epsilon_x + \sin\theta \epsilon_y)y_{p1} \end{pmatrix} \quad (2)$$

On the other hand, *P* is also determined as the intersection between the laser beam line and the (*XY*) plane of (*X<sub>p</sub>*,*Y<sub>p</sub>*,*Z<sub>p</sub>*) coordinate system in the reference coordinate. Laser beam equation is derived as

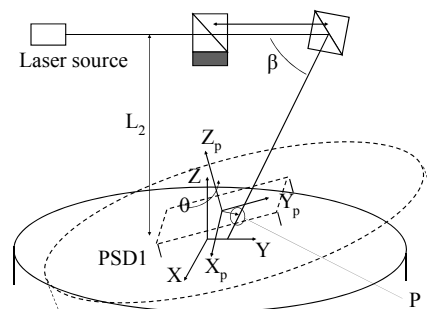


Fig. 3. PSD1 analysis.

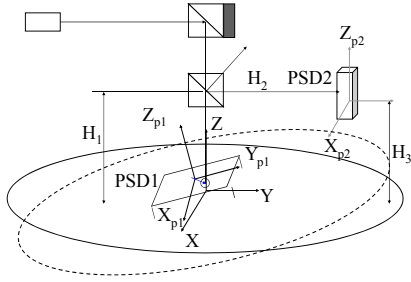


Fig. 4. Step 2 analysis.

$$\begin{cases} x = 0 \\ z = \tan \beta (y - a) \end{cases} \quad (3)$$

where  $\beta$  is the angle between the tilt laser beam line and the (XY) plane of the reference coordinate system. The equation of (XY) plane of  $(X_p, Y_p, Z_p)$  coordinate system is determined by the normal vector

$$\varepsilon_y x - \varepsilon_x y + z - \delta_z = 0 \quad (4)$$

Then we obtain the point by using the following equation:

$$\begin{aligned} x &= 0 \\ y &= \frac{\delta_z + a \tan \beta}{\tan \beta - \varepsilon_x} \\ z &= \frac{\delta_z + a \varepsilon_x}{1 - \varepsilon_x / \tan \beta} \end{aligned} \quad (5)$$

This intersecting point is transformed into the PSD1 coordinate, compared to the PSD1 signal  $(x_1, y_1)$ , and neglecting the high order terms we determine Eq. (6).

$$\begin{aligned} \tan \beta x_1 - a \tan \beta \sin \theta &= -\tan \beta \cos \theta \delta_x \\ &\quad - \tan \beta \sin \theta \delta_y + \sin \theta \delta_z + \varepsilon_x x_1 + a \tan \beta \cos \theta \varepsilon_z \\ \tan \beta y_1 - a \tan \beta \cos \theta &= -\tan \beta \sin \theta \delta_x \\ &\quad - \tan \beta \cos \theta \delta_y + \cos \theta \delta_z + \varepsilon_x y_1 - a \tan \beta \sin \theta \varepsilon_z \end{aligned} \quad (6)$$

### 2.2.2 Step2

In this step, the laser beam is split into two beams: vertical (VB) and horizontal (HB) beam. Through the beam splitter 2, the VB strikes PSD1 surface. The beam equation is the following:

$$x = 0, \quad y = 0 \quad (7)$$

The PSD plane equation is similar to Eq. (4) shown in the Step 1. The intersecting point is determined as:  $x=0, y=0, z=\delta_z$ . P is now on the z axis of reference coordinate. Similar to Step 1 we obtain

$$\begin{aligned} x_2 &= -\cos \theta \delta_x - \sin \theta \delta_y \\ y_2 &= \sin \theta \delta_x - \cos \theta \delta_y \end{aligned} \quad (8)$$

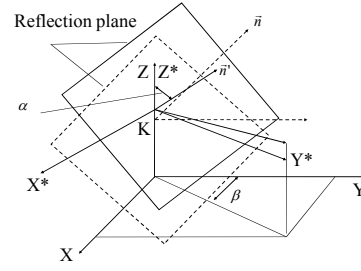


Fig. 5. Prism analysis.

For PSD2, we must determine the reflection surface, the normal vector and the reflection point. As shown in Fig. 5, vector  $n(0,1,1)$  is the normal vector in the PSD frame. After rotating with geometrical errors, the reflection plane is described as:

$$\begin{aligned} (\varepsilon_y - \sin \theta)x + (-\varepsilon_x + \cos \theta)y + (1 + \cos \theta \varepsilon_x + \sin \theta \varepsilon_y)z \\ + \sin \theta \delta_x - \cos \theta \delta_y - (1 + \cos \theta \varepsilon_x + \sin \theta \varepsilon_y)H_2 = 0 \end{aligned} \quad (9)$$

The reflection point is defined as the intersecting point between the incident laser beam line and the reflection plane.

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ H_1 + (\cos \theta \delta_y - \sin \theta \delta_x) / (1 + \cos \theta \varepsilon_x + \sin \theta \varepsilon_y) \end{pmatrix} \quad (10)$$

The PSD2 surface plane is defined as the (XZ) plane of the PSD2 coordinate and determined as:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \cos^2 \theta \varepsilon_y - \sin \theta - \cos \theta \varepsilon_z - \sin \theta \cos \theta \varepsilon_x \\ -\sin^2 \theta \varepsilon_x + \cos \theta - \sin \theta \varepsilon_z + \sin \theta \cos \theta \varepsilon_y \\ 2(\cos \theta \varepsilon_x + \sin \theta \varepsilon_y) \end{pmatrix} \quad (11)$$

The HB line lies on the plane made by the incident laser beam and the normal vector of the reflection plane and can be determined by using the HB vector:

$$\begin{aligned} (\cos \theta \varepsilon_z - \sin \theta)x + (\cos \theta - \sin \theta \varepsilon_z)y + (\cos \theta \varepsilon_x + \sin \theta \varepsilon_y)z \\ + \sin \theta \delta_x - \cos \theta \delta_y - H_2 = 0 \end{aligned} \quad (12)$$

The intersecting point is obtained by substituting the HB line equation into the plane equation:

$$\begin{aligned} x_{03} &= (\cos^2 \theta \varepsilon_y - \sin \theta - \cos \theta \varepsilon_z - \sin \theta \cos \theta \varepsilon_x)t \\ y_{03} &= (-\sin^2 \theta \varepsilon_x + \cos \theta - \sin \theta \varepsilon_z + \sin \theta \cos \theta \varepsilon_y)t \\ z_{03} &= \delta_z + \frac{\cos \theta \delta_y - \sin \theta \delta_x + H_1}{1 + \cos \theta \varepsilon_x + \sin \theta \varepsilon_y} + 2(\cos \theta \varepsilon_x + \sin \theta \varepsilon_y)t \\ t &= H_2 - H_1 + \frac{H_1 + \cos \theta \delta_y - \sin \theta \delta_x}{1 + \cos \theta \varepsilon_x + \sin \theta \varepsilon_y} \end{aligned} \quad (13)$$

Eq. (14) is obtained after the point K is transformed into the PSD2 coordinate system, compared to the PSD2 signal ( $x_3, z_3$ ), and neglecting the high-order terms.

$$\begin{aligned} x_3 &= -\cos\theta\delta_x - \sin\theta\delta_y + (\sin\theta H_1 - \sin\theta H_2 - x_3 \cos\theta)\varepsilon_x \\ &\quad + (\cos\theta H_2 - \cos\theta H_1 - x_3 \sin\theta)\varepsilon_y \\ z_3 &= -\sin\theta\delta_x + \cos\theta\delta_y + (\cos\theta H_2 - \cos\theta H_1 - z_3 \cos\theta)\varepsilon_x \\ &\quad + (\sin\theta H_2 - \sin\theta H_1 - z_3 \sin\theta)\varepsilon_y \end{aligned} \tag{14}$$

From Eqs. (6), (8), and (14), we can get the system of six equations.

$$B = AE \tag{15}$$

where

$$B = \begin{pmatrix} \tan\beta x_1 - a \tan\beta \sin\theta \\ \tan\beta y_1 - a \tan\beta \cos\theta \\ x_2 \\ y_2 \\ x_3 \\ z_3 \end{pmatrix}; \quad E = \begin{pmatrix} \delta_x \\ \delta_y \\ \delta_z \\ \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \end{pmatrix}$$

$$A = \begin{pmatrix} -\tan\beta \cos\theta & -\tan\beta \sin\theta & \sin\theta & x_1 \\ -\tan\beta \sin\theta & -\tan\beta \cos\theta & \cos\theta & y_1 \\ -\cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & -\cos\theta & 0 & 0 \\ -\cos\theta & -\sin\theta & 0 & (H_1 - H_2)\sin\theta - x_3 \cos\theta \\ -\sin\theta & \cos\theta & 0 & (H_2 - H_1)\cos\theta - z_3 \cos\theta \\ 0 & a \tan\beta \cos\theta \\ 0 & -a \tan\beta \sin\theta \\ 0 & 0 \\ 0 & 0 \\ (H_2 - H_1)\cos\theta - x_3 \sin\theta & 0 \\ (H_2 - H_1)\sin\theta - z_3 \sin\theta & 0 \end{pmatrix}$$

The singularity of Matrix A depends on  $H_2, H_1$ , and  $\tan\beta$ .

### 3. System configuration and measuring procedure

Deviations always exist between the actual position and orientation of the devices in the measurement system due to the mounting errors of each component that occur in the setup processes. It is similar with the definition of geometric errors; there are six components of mounting errors that exist for each component of the system and should be considered theoretically. However, by choosing the vertical laser beam as one of the reference coordinates, the mounting errors result from the misalignment of tilt laser beam and two PSDs, as shown in Fig. 6. As the vertical and tilt beams are set on the same point in the PSD plane, the (YZ) plane is defined as the plane that involves the vertical and tilt beams. The value of  $\beta$  is cali-

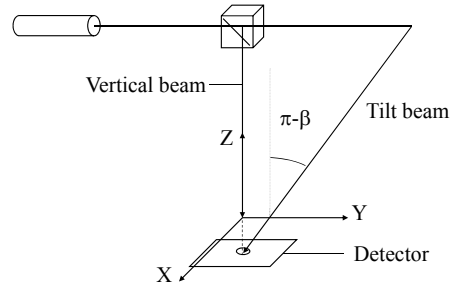


Fig. 6. Beam splitter and reflector adjustment.

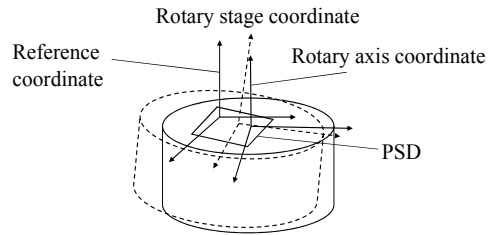


Fig. 7. Coordinate systems.

brated and also estimated with mounting error. The two beam splitters are adjusted to direct the transparent beam go in the same direction as the beam that was split by the beam splitter. The 90° deviation angle can be considered as the mounting error of the beam splitter. However, by using penta prisms with very high accuracy, this deviation is just approximately five acrsec, and can be neglected. There are six mounting error components between the PSD and rotary stage that should be considered as translational and rotational mounting errors. The transformation is expressed as follows:

$${}^rT_p = \begin{pmatrix} 1 & -ep_z & ep_y & a_1 + dp_x \\ ep_z & 1 & -ep_x & a_2 + dp_y \\ -ep_y & ep_x & 1 & a_3 + dp_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{16}$$

where ( $ep_x, ep_y, ep_z$ ) and ( $dp_x, dp_y, dp_z$ ) are the rotational and displacement mounting errors respectively. The original offset is represented by ( $a_1, a_2, a_3$ ). However, if the local coordinate system of the rotary stage is set at any angle of rotary stage surface and at any distance on z direction, then  $ep_z$  and  $dp_z$  can be set to zero.

On the other hand, there is a deviation between the rotary axis coordinate and reference coordinate which is defined similarly as the six geometrical errors. The transformation between these two coordinates is expressed as

$${}^R_rT = \begin{pmatrix} 1 & -er_z & er_y & d_1 + dr_x \\ er_z & 1 & -er_x & d_2 + dr_y \\ -er_y & er_x & 1 & d_3 + dr_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{17}$$

where  $(e_{r_x}, e_{r_y}, e_{r_z})$  and  $(d_{r_x}, d_{r_y}, d_{r_z})$  are the rotational and displacement mounting errors, respectively. The original offset is represented as  $(d_1, d_2, d_3)$ . Thus, the transformation from PSD to the reference coordinate is

$$R_n = {}^R_r T \cdot R \cdot {}^r_p T \quad (18)$$

The predefined values for translational and rotational mounting errors are set as  $\pm 100 \mu\text{m}$  and  $\pm 0.01$  rad respectively. An amount of  $\pm 0.01$  rad is added also into the value of angle  $\beta$ . Using the same derivation method with the transformation matrix as shown in Eq. (18), the estimated mounting error components with mounting errors are obtained and compared to the geometrical errors without mounting errors to check the magnitude of influence. The simulation, which is generated several times with different values of mounting errors shows that the influences of  $\beta$ , displacement mounting errors in the perpendicular direction of PSD normal vectors and tilt errors are crucial. The simulation results also show that the deviation of  $\beta$  should be controlled under 0.01 rad.

#### 4. Conclusion

This paper presented a simple and economical non-contact measurement system of the geometrical errors of a rotary axis using PSDs and a laser diode. The mathematical model and the system sensitivity analysis are described. The deviation of the angle  $\beta$  is within 0.01 rad. The simulation results represent the proposed system measure the six geometrical errors of a rotary axis with high accuracy.

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